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## STABILITY OF AN OSCILLATING BOUNDARY LAYER

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16. Abstract  Levchenko and Solov'yev (1972, 1974) have developed a stability theory for space periodic flows, assuming that the Floquet theory is applicable to partial differential equations. In the present paper, this approach is extended to unsteady periodic flows. A complete unsteady formulation of the stability problem is obtained, and the stability characteristics over an oscillation period are determined from the solution of the problem. Calculations carried out for an oscillating incompressible boundary layer on a plate showed that the boundary layer flow may be regarded as a locally parallel flow.			
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## STABILITY OF AN OSCILLATING BOUNDARY LAYER

V. Ya. Levchenko and A. S. Solov'yev

1. The problem of stability of nonstationary, and, in particular, periodic flows has long attracted the attention of the researchers because of its high theoretical and practical importance. Experimental observations of the transition to turbulence in some periodic flows revealed a number of interesting phenomena. In attempting to explain them, the researchers have turned to the theory of hydrodynamic stability. Early studies were based on the concept of quasistationary flow, which studied the stability of momentary velocity profiles [1, 2]. The quasistationary approach is apparently applicable if the change in velocity profile occurs fairly slowly in time, i.e., when the characteristic time for change in the main flow is considerably lower than the period of natural oscillations in the flow. However, the quasistationary approach is not satisfactory for many real situations, and could even lead to erroneous results [3]. Experiments on the transition to turbulence in oscillating boundary layers show that nonstationariness has a significant influence on the transition when the frequencies of the applied oscillations are on the order of unstable natural frequencies of the stationary boundary layer (Tollmin-Shlichting waves) [4-7]. There is a problem in applying the quasistationary model to these conditions. /10\*

Publications [8,9] have developed a nonlocal theory for stability of spatial-periodic flows which is based on a hypothesis of applicability of the Flock theory to partial differential equations. The correctness of this theory was confirmed by direct experiments [10]. In this work, the methods developed in [8,9] were extended to nonstationary periodic flows, i.e., an

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\*Numbers in right margin indicate pagination in original text.

attempt was made to set up and solve the problem of stability of these flows in a "complete, nonstationary situation," thus defining the stability characteristics for the oscillation period. Specific calculations were made for an oscillating boundary layer of incompressible liquid on a flat plate; the flow in the boundary layer in this case was considered to be "locally parallel."

A number of researchers in recent years, independently of each other, have abandoned the quasistationary approach and conducted studies on stability of oscillating flows in a "complete nonstationary formulation." Publication [3] solves the problem of stability of a layer of viscous liquid with free surface that is put into motion by the periodically oscillating lower boundary in relation to long-wave perturbations. Grosch and Salven [11] studied the stability of a Poiseuille modulated planar flow. von Kerczek and Davis [12] studied the stability of a finite Stokes layer between two parallel plates, one of which was brought into harmonic oscillation in its plane. These publications are also based on the /11 aforementioned hypothesis. The technique for computing the stability characteristics in these works varies and differs from that proposed by the authors. The authors do not know a solution to the problem of stability of an oscillating boundary layer in a "complete nonstationary formulation."

2. The behavior of small two-dimensional perturbations in a flat flow of viscous incompressible liquid is described by flow function  $\psi_1(x, y, t)$ , which satisfies the linearized vortex equation

$$\frac{\partial}{\partial t}(\Delta\psi') + \frac{\partial(\Delta\psi^0, \psi')}{\partial(x, y)} + \frac{\partial(\Delta\psi', \psi^0)}{\partial(x, y)} = \frac{1}{R} \Delta\Delta\psi', \quad \Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}. \quad (2.1)$$

Here  $\psi^0 = \psi^0(t, y)$  -- flow function of the main flow, which in the examined case is a function of time  $t$  and the coordinate  $y$ , perpendicular to the direction of the flow;  $R$  -- Reynolds number which is constructed using characteristic dimensional quantities of length and velocity. The function  $\psi^0$  does not depend on the longitudinal coordinate  $x$ , therefore, one can look for the solution

to equation (2.1) in the form

$$\psi^1(x, y, t) = e^{i\alpha x} \Psi(t, y). \quad (2.2)$$

We will examine the spatial growth in small perturbations, considering that  $\alpha = \alpha_r + i\alpha_i$  is a complex constant. In this case, the flow is stable if  $\alpha_i > 0$ , and unstable if  $\alpha_i < 0$ .

Assume that the main flow  $\psi^0(t, y)$  is periodic for  $t$  with period  $2\pi/\epsilon$ . Assuming that one can use the Flock theory for ordinary differential equations for the partial differential equation (2.1), we will look for the solution to (2.1) in the form

$$\Psi(t, y) = e^{i\omega t} \Phi(t, y), \quad (2.3)$$

where  $\omega$ --real constant, where spatial growth of the perturbations is examined, while  $\Phi(t, y)$ --periodic function  $t$  with the same period  $2\pi/\epsilon$  as the coefficients of equation (2.1). Some substantiations for this hypothesis are presented in publications [3, 11, 12]. By substituting into (2.1) functions  $\psi^0(t, y)$  and  $\Phi(t, y)$  in the form of infinite Fourier series for  $t$  with coefficients that depend on  $y$ ,

$$\psi^0(t, y) = \sum_{k=-\infty}^{\infty} h_k(y) e^{ik\epsilon t} + \sum_{k=0}^{\infty} h_k^*(y) e^{ik\epsilon t}, \quad (2.4)$$

$$\Phi(t, y) = \sum_{n=-\infty}^{\infty} \varphi_n(y) e^{in\epsilon t} \quad (2.5)$$

and equating the terms with the same exponents, we obtain an infinite connected system of linear, ordinary differential equations

$$L_n \varphi_n = \left( U - c + n \frac{g}{\alpha} \right) A_n - U'' \varphi_n - \frac{1}{i\alpha R} (\varphi_n^{IV} - 2\alpha^2 \varphi_n'' + \alpha^4 \varphi_n) = \quad (2.6)$$

$$= \sum_{k=1}^{\infty} \{ h_k^{III} \varphi_{n-k} + h_k^{*III} \varphi_{n+k} - h_k' A_{n-k} - h_k'' A_{n+k} \} \quad (n = 0, \pm 1, \dots).$$

Here the following designations are introduced

$$U = 2 \operatorname{Re} h'_0, \quad \omega = -\alpha c, \quad A_n = \varphi_n'' - \alpha^2 \varphi_n.$$

The asterisk designates the complexly-conjugate quantity, the apostrophe designates the derivative for  $y$ .

If the boundary conditions for  $\phi_n$  are uniform, then with assigned  $\omega$ ,  $R$  and  $\epsilon$  we have a problem for eigen values, and the flow is stable or unstable depending on the sign of  $\alpha_i$ . The  $\psi^0(t, y)$  flow which is studied for stability is known before the problem is solved, therefore, in the sums for  $k$  in expressions (3.4) and (2.6) one can be limited with the necessary accuracy to a finite number of terms in the Fourier series. The main difficulty in solving system (2.6) is that system (2.6) is coupled. We assume that if series (2.5) is also broken, i.e., we ignore in (2.6) the terms containing  $\varphi_{s+1}, \varphi_{s-1}, \varphi_{s+2}, \varphi_{s-2}, \dots$ , then, starting from a certain fairly large number  $n = s$ , where  $s$  is a whole positive number, the eigen values  $\alpha$  found will not depend on  $s$  essentially with the necessary accuracy. The system of equations that is thus obtained has a finite order, and can be integrated. In particular, with  $k = 1$ , it looks like

$$L_n \varphi_n = h_1^{III} \varphi_{n-1} + h_1^{*III} \varphi_{n+1} - h_1' A_{n-1} - h_1^{*'} A_{n+1}, \quad (n = -s, \dots, s). \quad (2.7)$$

The plan for integrating systems of this type is presented in detail in [8]. It is clear that the effectiveness of the described method for the solution depends significantly on the number of terms  $s$  taken into consideration in series (2.5). The use of the method is justified, if the value  $s$  is small, i.e., does not exceed two or three. The value  $s$  is selected while solving the problem and depends on the type of function  $\psi^0(t, y)$ . However, calculations show that with a reduction in the frequency parameter  $\epsilon$ , starting from certain values, one has to take into consideration a greater number of terms in the Fourier series. This results in

an undesirable increase in the order of the corresponding equation system, increases the time for numerical integration of the system on a computer, and the described direct method for solving the problem becomes ineffective.

On the other hand, when  $\epsilon$  is small, equation (2.1) contains coefficients which are slowly changing functions of time  $t$ , in that sense that their derivatives are proportional to the small parameter  $\epsilon$ . Asymptotic methods can be used in this case to solve equation (2.1). In this work, as in publication [9], the method of many scales is used in the process of constructing the asymptotic method to solve the problem in a complete nonstationary formulation.

3. In order to simplify the calculations, we will examine a case frequently encountered in the applied cases where  $\psi^0(t, y)$  looks like (in (2.4)  $k = 1$ )

$$\psi^0(t, y) = \psi_0(y) + \epsilon^0 \{h(y)e^{i\epsilon t} + h^*(y)e^{-i\epsilon t}\}, \quad (3.1)$$

where  $\psi_0 = 2 \operatorname{Re} h_0$ ,  $h_1 = \epsilon^0 h$ , while  $\epsilon^0$  -- a certain small amplitude parameter. The parameters  $\epsilon$  and  $\epsilon^0$  satisfy the conditions

$$\epsilon \ll 1, \epsilon^0 \ll 1. \quad (3.2)$$

We will introduce into (2.1) and (3.1) a new time scale  $T = \epsilon t$ , and, by using the formalism of the method of many scales, we will replace the derivative for  $t$  in (2.1) of the type  $\frac{\partial}{\partial t} \rightarrow \frac{\partial}{\partial t} + \epsilon \frac{\partial}{\partial T}$ .

In the resulting equation, the equations will not depend on  $x$  and  $t$ , consequently, one can search for the solution in the form

$$\psi^1(x, y, T, t) = \Phi(y, T) e^{i(\alpha x - \omega t)}, \quad (3.3).$$

where the following equation is obtained for the function  $\Phi(y, T)$

$$\left(-i\omega + \varepsilon \frac{\partial}{\partial T} + i\alpha \frac{\partial \psi^0}{\partial y}\right) \left(\frac{\partial^2}{\partial y^2} - \alpha^2\right) \Phi - i\alpha \frac{\partial^3 \psi^0}{\partial y^3} \Phi = \frac{1}{R} \left(\frac{\partial^2}{\partial y^2} - \alpha^2\right)^2 \Phi. \quad (3.4)$$

After this equation has been supplemented with uniform boundary conditions, we obtain a problem for eigen values to determine  $\alpha$ . The method for solving this type of problem is presented in detail in [9], therefore, we will present below only a brief description of the method and derive the main equations for the examined problem. /13

The coefficients of equation (3.4) are periodic functions  $T$  with period  $2\pi$ . We have a solution to equation (3.4) in the class of functions which are periodic for  $T$  with the same period  $2\pi$ , and require that the two small parameters  $\varepsilon^0$  and  $\varepsilon$  are proportional to each other with coefficient of proportionality  $k$ , i.e.,

$$\varepsilon^0 = k\varepsilon, \quad (3.5)$$

As indicated in [9] and discussed in detail in [13] in the example of a model equation, imposition of condition (3.5) is the only possibility of avoiding a quasistationary approximation. Decomposing  $\alpha$  and  $\Phi(t, y)$  into asymptotic series for  $\varepsilon$

$$\begin{aligned} \alpha &= \alpha_0 + \varepsilon \alpha_1 + \varepsilon^2 \alpha_2 + \varepsilon^3 \alpha_3 + \dots, \\ \Phi(T, y) &= \Phi_0(T, y) + \varepsilon^1 \Phi_1(T, y) + \varepsilon^2 \Phi_2(T, y) + \varepsilon^3 \Phi_3(T, y) + \dots, \end{aligned} \quad (3.6)$$

where all  $\Phi_i$  -- periodic functions with period  $2\pi$ , substituting (3.6) into (3.4) with regard for (3.5), and equating the terms with the same degrees of  $\varepsilon$ , we obtain a series of partial differential equations, from which the coefficients of expansion (3.6), constants  $\alpha_i$  and function  $\Phi_i$  are successively determined. The terms of the zero order yield the equation

$$L\Phi_0 = 0, \quad (3.7)$$

where  $L \equiv (-i\omega + i\alpha_0 \psi_0') \left(\frac{\partial^2}{\partial y^2} - \alpha_0^2\right) - i\alpha_0 \psi_0''' - \frac{1}{R} \left(\frac{\partial^2}{\partial y^2} - \alpha_0^2\right)^2$  -- operator of Orr-Sommerfield. One can search for the solution to this equation



in the form

$$\Phi_0(T, y) = \phi_0(y) B(T). \quad (3.8)$$

The eigen value  $\alpha_0$  and function  $\phi_0(y)$  are found from the Orr-Sommerfeld equation uniform boundary conditions for  $\phi_0$ , while the  $B(T)$  function is determined from a nonuniform equation obtained by leveling the terms with  $\varepsilon^1$

$$L\Phi_1 = -kB(T)e^{iT}F_{11}(y) - kB(T)e^{-iT}F_{12}(y) - \frac{dB}{dT}F_{13}(y) - \alpha_1 B(T)F_{14}(y), \quad (3.9)$$

where

$$\begin{aligned} F_{11}(y) &= i\alpha_0(h'\Delta\phi_0 - h'''\phi_0), \\ F_{12}(y) &= i\alpha_0(h''\Delta\phi_0 - h''''\phi_0), \\ F_{13}(y) &= \Delta\phi_0, \quad F_{14}(y) = -2\alpha_0(-i\omega + i\alpha_0\psi_0') - i\psi_0'''\phi_0 + \left(i\psi_0' + \frac{4\alpha_0}{R}\right)\Delta\phi_0, \\ \Delta &= \frac{\partial^2}{\partial y^2} - \alpha_0^2. \end{aligned} \quad (3.10)$$

This equation, supplemented by uniform boundary conditions for  $\phi_1$  has a solution only when the condition of orthogonality is fulfilled

$$\begin{aligned} \frac{dB}{dT} \int_0^\infty \phi_0^+ F_{13} dy + ke^{iT}B(T) \int_0^\infty \phi_0^+ F_{11} dy + ke^{-iT}B(T) \int_0^\infty \phi_0^+ F_{12} dy + \alpha_1 B(T) \times \\ \times \int_0^\infty \phi_0^+ F_{14} dy = 0. \end{aligned} \quad (3.11)$$

The solutions to the equation, conjugate to the Orr-Sommerfeld equation are designated here by  $\phi_0^+$ . By using the requirement for periodicity of the  $B(T)$  function, it is easy to show [9] that  $\alpha_1 \equiv 0$ , while the  $B(T)$  function looks like

$$B(T) = B_0(k) \exp[ik(\Gamma_{11}e^{iT} - \Gamma_{12}e^{-iT})], \quad (3.12)$$

where

$$\Gamma_{1m} = \frac{\int_0^\infty \phi_0^+ F_{1m} dy}{\int_0^\infty \phi_0^+ F_{13} dy} \quad (m = 1, 2),$$

while  $B_0(k)$ --constant that is not defined in framework of the linear theory. We search for the solution to equation (3.9) in the form

$$\Phi_1(y, T) = B(T) [e^{iT} \varphi_1(y) + e^{-iT} \varphi_2(y)] + B_1(T) \varphi_0(y), \quad (3.13)$$

where the functions  $\phi_1$  and  $\phi_2$  are found from the nonuniform Orr-Sommerfield equations:

$$L\varphi_m = k[-F_{1m}(y) + \Gamma_{1m} F_{13}(y)] \quad (m=1, 2) \quad (3.14)$$

with uniform boundary conditions for  $\phi_m$ . In order to determine the function  $B_1(T)$  it is necessary to examine the terms with  $\varepsilon^2$ . The equation for  $\phi_2$  looks like

$$\begin{aligned} L\Phi_2 = & -kB_1(T)e^{iT}F_{11}(y) - kB_1(T)e^{-iT}F_{12}(y) - \frac{dB_1}{dT}F_{13}(y) - B(T)e^{2iT}F_{21}(y) - \\ & -B(T)e^{iT}F_{22}(y) - B(T)F_{23}(y) - B(T)e^{-iT}F_{24}(y) - \\ & -B(T)e^{-2iT}F_{25}(y) - \alpha_2 B(T)F_{14}(y). \end{aligned} \quad (3.15)$$

Here

$$\begin{aligned} F_{21} &= k(F_{11}^{(1)} - \Gamma_{11}F_{13}^{(1)}), \quad F_{22} = iF_{13}^{(1)}, \\ F_{23} &= k(F_{12}^{(1)} - \Gamma_{12}F_{13}^{(1)} + F_{11}^{(2)} - \Gamma_{11}F_{13}^{(2)}), \\ F_{24} &= -iF_{13}^{(2)}, \quad F_{25} = k(F_{12}^{(2)} - \Gamma_{12}F_{13}^{(2)}). \end{aligned} \quad (3.16)$$

From the condition of orthogonality,  $B_1(T)$  is determined, while the requirement for periodicity yields

$$\alpha_2 = - \frac{\int_0^\infty \varphi_0^+ F_{23} dy}{\int_0^\infty \varphi_0^+ F_{14} dy}. \quad (3.17)$$

Continuing the described algorithm, we obtain  $\alpha_3$ , etc. quite

analogously.

$$\alpha_3 = - \frac{\int_0^\infty \varphi_0^+ F_{111} dy}{\int_0^\infty \varphi_0^+ F_{14} dy}, \quad (3.18)$$

where

$$F_{111} = k(F_{12}^{(3)} - \Gamma_{12}F_{13}^{(3)} + F_{11}^{(4)} - \Gamma_{11}F_{13}^{(4)}). \quad (3.19)$$

We note that in order to obtain the functions  $F_{1n}^{(m)}$ , in (3.16) and (3.19), it is necessary to replace the function  $\phi_0$  by the function  $\phi_m$  ( $m = 1, \dots, 4$ ). The functions  $\phi_1$  and  $\phi_2$  are found from (3.14), while  $\phi_3$  and  $\phi_4$  are determined from the equations

$$\begin{aligned} L\varphi_3 &= -\frac{1}{2}ik\Gamma_{21}(-F_{12} + \Gamma_{12}F_{13}) - (F_{22} + \Gamma_{22}F_{13}), \\ L\varphi_4 &= \frac{1}{2}ik\Gamma_{25}(-F_{11} + \Gamma_{11}F_{13}) - (F_{24} + \Gamma_{24}F_{13}) \end{aligned} \quad (3.20)$$

with the corresponding uniform boundary conditions. Here

$$\Gamma_{2n} = \frac{\int_0^\infty \varphi_0^+ F_{2n} dy}{\int_0^\infty \varphi_0^+ F_{13} dy} \quad (n = 1, \dots, 5). \quad (3.21)$$

Thus, by analyzing (3.18) - (3.20) with regard for (3.14) and (3.16) and (3.21), and taking (3.6) into consideration with accuracy to terms on the order of  $\varepsilon^3$ , we find

$$\alpha = \alpha_0 + \varepsilon^2 k^2 C_0 + \varepsilon^3 (k^2 C_1 + k^4 C_2) + O(\varepsilon^4). \quad (3.22)$$

Here  $C_n$  ( $n = 0, \dots, 2$ ) are known constants which depend on  $\alpha_0$ ,  $R$ ,  $\omega$

and do not depend on  $\varepsilon$  and  $k$ .

4. The mode of flow in the oscillating boundary layer with rate of the external flow  $U = 1 + \varepsilon^0 \cos \varepsilon t$  is characterized by the parameter  $\zeta = \frac{\bar{x}\bar{\varepsilon}}{\bar{U}_m}$ . The dash notes dimensional quantities.

Here  $\bar{x}$ --longitudinal coordinate,  $\bar{U}_m$ --average rate of external flow,  $\bar{\varepsilon}$ --cyclic frequency of applied oscillations. With  $\zeta < 1$ , the flow is considered to be quasistationary, with  $\zeta > 10$  it is high-frequency; the values  $1 < \zeta < 10$  correspond to the intermediate mode. Ackerberg and Phillips obtained asymptotic solution to the equations of the oscillating boundary layer on a flat plate for  $\zeta \rightarrow \infty$  which is correct to values  $\zeta > 4$  [14]. This solution was used in calculating the stability characteristics.

Figure 1 illustrates the neutral curves, figure 2 shows the curves for the coefficients of perturbation amplification with fixed frequency parameter  $F = \frac{\omega v}{\bar{U}_m^2} = 110 \cdot 10^{-6}$ , where  $\omega$ --dimensional frequency of perturbation,  $v$ --viscosity, while the amplification coefficients are designated by  $\ln a = - \int_{R_0}^R \alpha_i \delta dR$ . Curves 1 were computed for a stationary boundary layer (Blasius profile). Curves 2 are the results of calculations for the amplitude of applied oscillations  $\varepsilon^0 = 0.1$ , and dimensionless frequency  $F_0 = \frac{\bar{\varepsilon} v}{\bar{U}_m^2} = 45 \cdot 10^{-6}$ .

The calculations were made by both aforementioned methods. In that region of frequencies where  $\varepsilon$  is fairly small and it can be viewed as a small parameter of the problem, the asymptotic calculation method was used (we note that  $\varepsilon = F_0 R$  diminishes with a decline in  $R$  and  $F_0$ ;  $R$ --Reynolds number constructed using the characteristic dimension  $\delta = \sqrt{\frac{\nu x}{\bar{U}_m}}$ ). The neutral curve 2 in

/16

figure 1 was calculated both by the direct method described in section 2 (two terms in the Fourier expansion were considered,  $s = 2$ ), and by the asymptotic method, using formula (3.22) (the

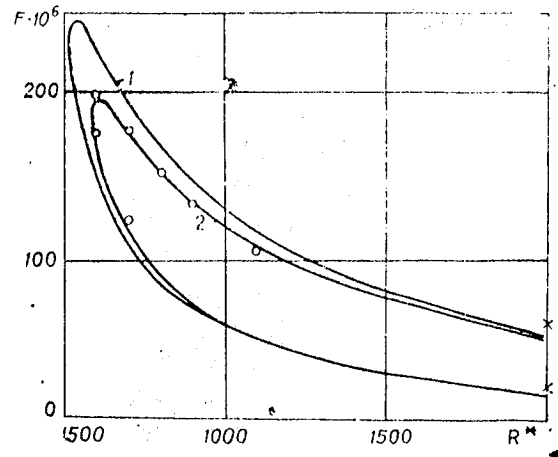


Figure 1.

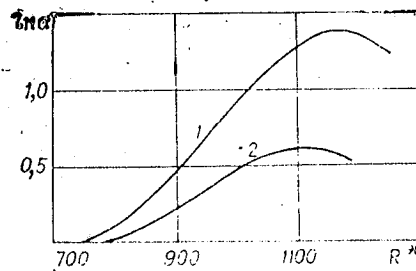


Figure 2.

corresponding points of the neutral curve are designated by light circles). The results show the good agreement of the presented methods.

The dimensionless frequency  $F_0 = 45 \times 10^{-6}$  was selected to be close to the eigen frequency of the stationary boundary layer observed near the point of transition to turbulence [15].

When acoustic oscillations are applied (amplitudes of the oscillations are small) with frequencies that coincide with unstable eigen frequencies of the stationary boundary layer that correspond to the internal region of the neutral stability curve, a significant increase was observed in the experiments in the Reynolds transition number, i.e., destabilization of the boundary

layer [4-7]. Since in boundary layer type flows, the transition to turbulence is due to the instability in the laminar flow in relation to small perturbations, one should expect the destabilizing influence of oscillations in the external flow on the stability characteristics of the boundary layer. However, the calculation results were unexpected and seemingly contradictory to the known experimental facts. For the value of the amplitude of oscillation in external flow  $\epsilon^0 = 0.05$ , the calculations indicated that the influence of nonstationary flow on the characteristics of its stability is essentially missing. With an increase in the modulation amplitude of the external flow, the flow stabilizes in the boundary layer: the curve of neutral stability for  $\epsilon^0 = 0.1$  (curve 2 in fig. 1) is shifted into the region of large Reynolds numbers; the slowing down of growth in perturbation is visible in this case in figure 2 (curve 2).

Direct experimental verification partially confirmed the correctness of this theory: with low amplitudes of applied oscillations, no change was noted in the stability characteristics of the boundary layer; destabilization in the oscillating boundary layer observed in the experiments is not explained by the increased flow stability, but by other reasons [16].

Stabilization of the boundary layer with "high-frequency" oscillations in the external flow with fairly high amplitude has not yet been experimentally confirmed; there are no experimental data for these conditions. The possibility of stabilization has basically been excluded. Stabilization was experimentally observed for flow in the boundary layer under the influence of sound waves transverse to the direction of the flow. The stabilizing effect of modulation on the flow of other types is known [11]. The results of this work can be qualitatively compared with the results of von Kerczek and Davis [12]. With high-frequency oscillations, all the influence of nonstationariness on flow in the boundary layer is manifest in a narrow band near the surface, the so-called Stokes layer. The calculations of von Kerczek and

Davis indicated very high stability of the finite Stokes layer, and the stability improves with an increase in the oscillation amplitude.

The experiments of Obremski and Fejer [17] studied the effect of low-frequency oscillations on the transition in the boundary layer. Destabilization was observed in the flow with values of parameter  $Re_{NS} = \frac{e^0}{F_0}$ , larger than  $2,6 \times 10^4$ . With  $Re_{NS} < 2,6 \times 10^4$ , there is no influence of nonstationary flow on the transition in these experiments. In the calculations, whose results are discussed above, the  $Re_{NS}$  values did not exceed 2500. In figure 1, the cross designates the neutral points that were calculated for  $Re_{NS} = 10^5$ ,  $F_0 = 3 \times 10^{-6}$ , which corresponds roughly to mode No. 2 in the experiments [17], in which significant reduction was observed in the Reynolds transition number. Unfortunately, we did not succeed in advancing into the region of smaller Reynolds numbers with the employed velocity profiles or constructing the entire neutral curve, since the values of the  $\epsilon$  parameter in this case became smaller than 4. However, there was a noticeable expansion in the range of unstable frequencies, which indicates destabilization of the flow. Of course, it is impossible to draw definite conclusions from two points, but it is possible that in contrast to the case of "high frequencies", destabilization of the flow at low oscillation frequencies of the external flow is explained by increased instability of the nonstationary velocity profile. Further studies of this question are needed.

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